

# Symmetry energy in cold dense matter

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We calculate the symmetry energy in cold dense matter both in the normal quark phase and in the 2-color superconductor (2SC) phase. For the normal phase, the thermodynamic potential is calculated by using hard dense loop (HDL) resummation to leading order, where the dominant contribution comes from the longitudinal gluon rest mass. The effect of gluonic interaction to the symmetry energy, obtained from the thermodynamic potential, was found to be small. In the 2SC phase, the non-perturbative BCS pairing gives enhanced symmetry energy as the gapped states are forced to be in the common Fermi sea reducing the number of available quarks that can contribute to the asymmetry. We used high density effective field theory to estimate the contribution of gluon interaction to the symmetry energy. Among the gluon rest masses in 2SC phase, only the Meissner mass has iso-spin dependence although the magnitude is much smaller than the Debye mass. As the iso-spin dependence of gluon rest masses is even smaller than the case in the normal phase, we expect that the contribution of gluonic interaction to the symmetry energy in the 2SC phase will be minimal. The different value of symmetry energy in each phase will lead to different prediction for the particle yields in heavy ion collision experiment.

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## I. INTRODUCTION

There is a world wide interest in symmetry energy as it provides a key to understanding the iso-spin asymmetric property of the nuclear matter. Furthermore, related topics range from rare isotopes at low energy regime to neutron star core at high energy regime [1, 2]. The value of the symmetry energy at normal nuclear matter can be inferred from either the binding energy of semi empirical mass formula [3] or experimental information from isotopes or heavy ion collision [2]. However, the value at higher density is still under debate. Some model calculations including iso-spin dependent interaction channel (NL $\rho\delta$ ) [4] predict a stiff symmetry energy as density becomes higher, which can cause less favourable condition for the neutron and subsequently decreased iso-spin density.

The isospin asymmetric matter could persist longer in a heavy ion collision if the high density phase allows for a quark and hadron mixed phase. In such cases, it was found that the iso-spin density can remain high even if the hadronic part of the symmetry energy rises stiffly at high density [5–7]. This will lead to the enhanced iso-spin rich resonances and subsequently to the increased iso-spin asymmetric decays. This means that the particle yields can be modified not only by the nuclear symmetry energy but also by the quark part. As representative studies, the MIT bag model was used to calculate the symmetry energy when the quark phase is normal [5, 6], and additional color superconducting effect was taken into account through an NJL type model calculation [7]. Also the confined iso-spin density dependent mass (CIDDMM) model was used in Ref. [8] to obtain the quark matter symmetry energy and related influence in the quark star.

In the extremely dense condition, one should consider the equation of state for quark matter. The equation of state for non-interacting quarks with up to three flavors was first introduced in Ref. [9] in relation to the hypothetical quark stars. Due to asymptotic freedom, methods based on perturbative quantum chromodynamics (QCD) can be applied to calculate physical observables in extreme condition. If there exists some hard excitation scale in the matter, soft excitations can have infinite number of hard loop corrections that are not suppressed by the order of coupling and that can be resummed to an equivalent order [10–12]. Considering the cold normal dense matter first, in-medium quark excitation scale is hard due to its large chemical potential, whereas the gluon excitation scale is mainly soft. Therefore, all equivalent gluonic excitations which contains hard dense quark loop corrections should be resummed [13]. Secondly, when temperature becomes even lower, all the fermions will be confined below their Fermi sea without large fluctuation. As the momentum fluctuation scales to a small value, most of the operators can be absorbed into the energy term or neglected. On the other hand, the four-fermion interaction with opposite momenta becomes relevant [14–17]. If the interaction is attractive, it is natural for the fermions to form a condensate which leads to superconductivity: BCS paring [18]. For QCD matter, as the quarks in the color anti-triplet channel are attractive, color superconductivity arises naturally with large gap size [19–23]. At the density regime where the quark-hadron mixed phase may exist, 2-color superconductivity (2SC) will be favored as there is a mismatch of Fermi momentum between light quarks and strange quark [24].

In this work, we calculated the symmetry energy in cold dense matter using QCD. The density region being considered lies between 3 to 5 times the nuclear matter density  $\rho_0 = 0.16 \text{ fm}^{-3}$ , which could be reached in heavy ion collision experiment. For the normal phase, we use the hard dense loop (HDL) resummation for the gluonic interaction. Thermodynamic potential is obtained to leading order in HDL perturbation, which is an extension/modifying of previously reported works [25–27]. The symmetry energy has been calculated from the thermodynamic potential. For color superconductor phase, 2SC phase is considered. Without perturbative gluonic interaction, we adopted thermodynamic potential in 2SC phase from Ref. [28, 29] and calculated symmetry energy. To estimate the contribution of perturbative gluonic interaction, iso-spin dependence of gluon rest masses is calculated by using high density effective theory (HDET) [30–33].

This paper is organized as follows: in Sec. II, brief summary for quark and gluon modification by HDL resummation, the thermodynamic potential and the symmetry energy in the normal phase are presented. In Sec. III, the gluon rest masses in iso-spin asymmetric 2SC phase is calculated by following HDET and its contribution to the symmetry energy is estimated. Discussion and conclusions are given in Sec. IV.

## II. SYMMETRY ENERGY AT LOW TEMPERATURE

### A. Nuclear symmetry energy from equation of state

Discussion for symmetry energy can start from a finite nuclei with  $A$  nucleons [3]. The Bethe-Weizsäcker formula for the nuclear binding energy is given as

$$\begin{aligned} m_{\text{tot}} &= Nm_n + Zm_p - E_B/c^2, \\ E_B &= a_V A - a_S A^{\frac{2}{3}} - a_C(Z(Z-1))A^{-\frac{1}{3}} - a_A I^2 A + \delta(A, Z), \end{aligned} \quad (1)$$

where  $I = (N - Z)/A$ . The fourth term in  $E_B$  accounts for the total shifted energy due to the neutron number excess. The coefficient of fourth term  $a_A$  represents the shifted energy per nucleon.

When the atomic number  $A$  becomes large, one can approach a limit where parameters describing the system become continuous and a mean field type of approach appropriate. In such a system, the symmetry energy can be defined from energy per nucleon number:

$$\frac{E(\rho_N, I)}{A} \equiv \bar{E}(\rho_N, I) = E_0(\rho_N) + E_{\text{sym}}(\rho_N)I^2 + O(I^4) + \dots, \quad (2)$$

$$E_{\text{sym}}(\rho_N) = \frac{1}{2!} \frac{\partial^2}{\partial I^2} \bar{E}(\rho_N, I), \quad (3)$$

where  $A$  is the atomic number,  $\rho_N$  the nuclear medium density and  $I$  the asymmetric parameter  $I = (N - Z)/A \rightarrow (\rho_n - \rho_p)/(\rho_n + \rho_p)$ . The neutron and proton densities can be found as  $\rho_n = \frac{1}{2}\rho_N(1 + I)$ ,  $\rho_p = \frac{1}{2}\rho_N(1 - I)$  respectively.

### B. HDL resummed thermodynamic potential

In this section, we work in the imaginary time formalism ( $t \rightarrow -i\tau$ ,  $0 \leq \tau \leq \beta$ ) following Ref. [34]. The partition function  $\mathcal{Z}_\Omega$  can be obtained in QCD degree of freedom:

$$\mathcal{Z}_\Omega = \text{Tr} \exp \left[ -\beta(\hat{H} - \vec{\mu} \cdot \vec{N}) \right] = \int \mathcal{D}(\bar{\psi}, \psi, A, \eta) \exp \left[ -\int_0^\beta d\tau \int d^3x \mathcal{L}_E(\bar{\psi}, \psi, A, \eta) \right], \quad (4)$$

where the Euclidean QCD Lagrangian  $\mathcal{L}_E$  is given as

$$\begin{aligned} \mathcal{L}_E &= \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2\xi} (\partial_\mu A_\mu^a)^2 + \bar{\eta}^a (\partial^2 \delta_{ab} + g f_{abc} \partial_\mu A_\mu^c) \eta^b \\ &+ \sum_f^{n_f} \left[ \bar{\psi}_f^\dagger \partial_\tau \psi_f + \bar{\psi}_f (-i\gamma^i \partial_i + m_f) \psi_f - \mu_f \bar{\psi}_f^\dagger \psi_f - g \bar{\psi}_f A \psi_f \right], \end{aligned} \quad (5)$$

where  $\eta^a$  is ghost fields and the subscript  $f$  represents the quark flavor. From Eq. (4) and Eq. (5), the free propagators of fields can be obtained as

$$S_F(P) = \frac{m_f - \not{P}}{-(i\omega_n + \mu_f)^2 + \vec{p}^2 + m_f^2}, \quad (6)$$

$$\begin{aligned} D_{F,\mu\nu}^{ab}(Q) &= \delta^{ab} \left[ \frac{1}{Q^2} \left( \delta_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} \right) + \frac{\xi}{Q^2} \frac{Q_\mu Q_\nu}{Q^2} \right] \\ &= \delta^{ab} \left[ \frac{1}{Q^2} (P_{\mu\nu}^L + P_{\mu\nu}^T) + \frac{\xi}{Q^2} \frac{Q_\mu Q_\nu}{Q^2} \right], \end{aligned} \quad (7)$$

where  $S_F(P)$  is the free quark propagator,  $D_{F,\mu\nu}^{ab}(Q)$  the free gluon propagator and  $\xi$  the gauge fixing term. The longitudinal and transverse projection operators are defined as

$$\begin{aligned} P_{ij}^T &= \delta_{ij} - \hat{q}_i \hat{q}_j, \quad P_{44}^T = P_{4i}^T = 0, \\ P_{\mu\nu}^L &= \delta_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} - P_{\mu\nu}^T. \end{aligned} \quad (8)$$

In the imaginary formalism, the energy integration at each loop changes into a sum over discrete Matsubara frequencies:

$$\int \frac{d^4 k}{(2\pi)^4} \Rightarrow \int \frac{d^4 K}{(2\pi)^4} = T \sum_n \int \frac{d^3 k}{(2\pi)^3}, \quad (9)$$

where  $T \sum_n$  is over  $i\omega_n = i(2n)\pi T$  and  $i\tilde{\omega}_n = i(2n+1)\pi T + \mu_f$  for bosons and fermions, respectively.

### 1. HDL resummed propagator

Before calculating the gluonic interaction to the free energy, we summarize the medium modification of the propagator. First, we consider the gluon case. Suppressing light quark masses and soft external momenta ( $m_f/\mu_f \simeq 0$ ,  $K_\mu - Q_\mu \simeq K_\mu$ ) against the hard scale of internal loop ( $K_\mu \sim T, \mu_f$ ), the gluon polarization tensor in the dense matter can be calculated as [10–12]

$$\begin{aligned} \Pi_{\mu\nu}^{ab}(Q) &= g^2 \delta^{ab} \int \frac{d^4 K}{(2\pi)^4} \text{Tr} [\gamma_\mu S_F(K) \gamma_\nu S_F(K-Q)] \\ &= m^2 \delta^{ab} \int \frac{d\Omega}{4\pi} \left( \delta_{\mu 4} \delta_{\nu 4} + \hat{K}_\mu \hat{K}_\nu \frac{i\omega}{Q \cdot \hat{K}} \right), \end{aligned} \quad (10)$$

where  $\hat{K}_\mu = (-i, \hat{k} = \vec{k}/|\vec{k}|)$  is light-like four-vector,  $Q_\mu = (-\omega, \vec{q})$  is the Euclidean gluon external momentum and  $m^2$  is defined as

$$m^2 = \frac{1}{3} g^2 T^2 \left( C_A + \frac{1}{2} n_f \right) + \frac{1}{2} g^2 \sum_f \frac{\mu_f^2}{\pi^2}. \quad (11)$$

At low temperature, only HDL contribution becomes relevant:  $m^2 \Rightarrow \frac{1}{2} g^2 \sum_f \mu_f^2 / \pi^2 \sim g^2 \mu_f^2$  [13]. The polarization tensor can be decomposed into loop structures or polarizations as follows:

$$\Pi_{\mu\nu}^{ab}(Q) = \delta^{ab} (\Pi_{\mu\nu}(Q)|_{\text{q-h}} + \Pi_{\mu\nu}(Q)|_{\text{q-a}}) \quad (12)$$

$$= \delta^{ab} (\delta \Pi^L(Q) P_{\mu\nu}^L + \delta \Pi^T(Q) P_{\mu\nu}^T). \quad (13)$$

First,  $\Pi_{\mu\nu}(Q)|_{\text{q-h}}$  and  $\Pi_{\mu\nu}(Q)|_{\text{q-a}}$  represents the quark-hole contribution and the quark-antiquark contribution respectively [35, 36]:

$$\Pi_{\mu\nu}(Q)|_{\text{q-h}} = m^2 \int \frac{d\Omega}{4\pi} \hat{K}_\mu \hat{K}_\nu \left( -1 + \frac{i\omega}{Q \cdot \hat{K}} \right), \quad (14)$$

$$\Pi_{\mu\nu}(Q)|_{\text{q-a}} = m^2 \int \frac{d\Omega}{4\pi} \left( \delta_{\mu 4} \delta_{\nu 4} + \hat{K}_\mu \hat{K}_\nu \right). \quad (15)$$

Second, the polarization  $\delta \Pi^L(Q)$  and  $\delta \Pi^T(Q)$  can be obtained as follows [34]:

$$\delta \Pi^L(Q) = -2m^2 \frac{Q^2}{q^2} Q_1 \left( \frac{i\omega}{q} \right), \quad (16)$$

$$\delta \Pi^T(Q) = m^2 \left( \frac{i\omega}{q} \right) \left[ \left( 1 - \left( \frac{i\omega}{q} \right)^2 \right) Q_0 \left( \frac{i\omega}{q} \right) + \left( \frac{i\omega}{q} \right) \right], \quad (17)$$

where  $Q_0(x) = \frac{1}{2} \ln [(x+1)/(x-1)]$ ,  $Q_1(x) = xQ_0(x) - 1$ . Here one can find that the longitudinal rest mass comes mainly from the quark-hole contribution.

For the gluon propagator, if the external gluon momentum is soft ( $Q \simeq g\mu$ ), all diagram where HDL is inserted as 1PI self energy scales in the same order as the bare gluon propagator ( $1/Q^2 \sim 1/(g^2 \mu_f^2)$ ). The resummed gluon propagator is given as follows [34]:

$$*D_{\mu\nu}^{ab}(Q) = \delta^{ab} \left( \frac{1}{Q^2 + \delta \Pi^L(Q)} P_{\mu\nu}^L + \frac{1}{Q^2 + \delta \Pi^T(Q)} P_{\mu\nu}^T + \frac{\xi}{Q^2} \frac{Q_\mu Q_\nu}{Q^2} \right). \quad (18)$$

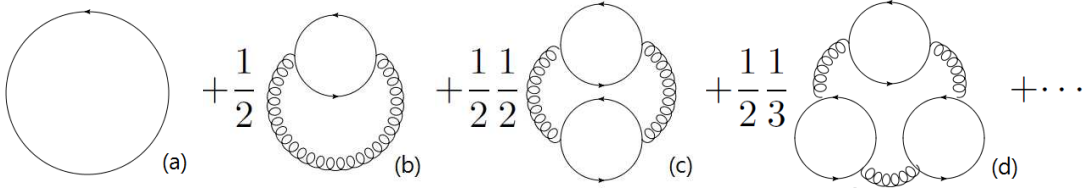


FIG. 1. Diagrammatic description for  $\ln \mathcal{Z}_{\Omega_{q,0}} + \ln \mathcal{Z}_{\Omega_{g,\text{HDL}}}$ . Each diagram is presented with its symmetric factor.

Next we consider the quark case. If the external momenta can be neglected against the hard scale of internal loop, the quark self energy can be calculated as [34, 37]

$$\begin{aligned} \Sigma(P) &= -g^2 \tau^a \tau^b \int \frac{d^4 K}{(2\pi)^4} \gamma_\mu S_F(P-K) \gamma_\nu D_{\mu\nu}^{ab}(K) \\ &= m_f^2 \int \frac{d\Omega}{4\pi} \frac{\hat{K}}{P \cdot \hat{K}}, \end{aligned} \quad (19)$$

where  $P_\mu = (-\tilde{\omega}, \vec{p})$  is the Euclidean quark external momentum,  $\tau^a$  is the fundamental representation of  $\text{SU}(3)$  generator normalized as  $\text{Tr}[\tau^a \tau^b] = \frac{1}{2} \delta^{ab}$  and  $m_f^2$  is defined as

$$m_f^2 = \frac{1}{8} g^2 C_F \left( T^2 + \frac{\mu_f^2}{\pi^2} \right), \quad (20)$$

where  $C_F = (N_c^2 - 1)/(2N_c)$ . Again, only HDL contribution becomes relevant at low temperature:  $m_f^2 \Rightarrow \frac{1}{8} g^2 C_F \mu_f^2 / \pi^2 \sim g^2 \mu_f^2$  [37]. By similar argument with the gluon case, the resummed quark propagator can be given as follows [34]:

$$*S(P) = \frac{1}{\not{P} + \Sigma(P)}, \quad (21)$$

where

$$\begin{aligned} \not{P} + \Sigma(P) &= A_4 \gamma_4 + A_s \vec{\gamma} \cdot \hat{p}, \\ A_4 &= i \left( i\tilde{\omega} - \frac{m_f^2}{|\vec{p}|} Q_0 \left( \frac{i\tilde{\omega}}{|\vec{p}|} \right) \right) \\ A_s &= |\vec{p}| + \frac{m_f^2}{|\vec{p}|} \left[ 1 - \frac{i\tilde{\omega}}{|\vec{p}|} Q_0 \left( \frac{i\tilde{\omega}}{|\vec{p}|} \right) \right]. \end{aligned} \quad (22)$$

## 2. Calculating the thermodynamic potential with HDL resummation

The free energy can be written as

$$\Omega(\mu) = \langle \hat{H} \rangle - \vec{\mu} \cdot \langle \vec{N} \rangle = -\frac{1}{\beta} \ln \mathcal{Z}_\Omega, \quad (23)$$

where  $\ln \mathcal{Z}_\Omega$  corresponds to the sum of all connected ring diagrams. If the interactions are weak enough, one can calculate elements in  $\ln \mathcal{Z}_\Omega$  by perturbative method. However, as the soft excitations in the hot/dense matter have non-perturbative loop corrections, this quantity should be obtained by accounting for all equivalent contribution through resummation. In the low temperature and dense limit ( $T \simeq g\mu \ll \mu$ ), gluonic excitation will be mainly soft ( $Q \sim T \simeq g\mu$ ). In this limit, the physically relevant leading strong interaction contribution comes from the leading quark loop (Fig. 1(a)) and from HDL resummed gluon ring diagrams (Fig. 1(b)-(d)).

The simplest diagram is Fig. 1(b): a soft gluon ring with hard quark loop correction which can also be seen as a hard quark ring with soft gluon loop correction. As the gluon line is soft, equivalent HDL corrections should be accounted for as a sum of infinite series (Fig. 1(c)-(d)): HDL resummation of gluon ring diagrams.

Meanwhile, this type of resummation may not be suitable for quarks as the physically relevant quark excitations at low temperature and dense limit ( $|\vec{p}| \sim \mu_f$ ) can not have HDL corrections [35]. In this limit, gluon loop should carry soft momentum so that the correction will be suppressed by higher orders in  $\alpha_s$ . However, as there can exist suitable intermediate state for HDL resummation among the loop integration of the quark ring trace, we will adopt the resummation result reported in Refs. [26, 27].

In this work, we consider 2-flavor light quark matter for simplicity. Moreover near the beginning of the quark phase, the baryon density ranges from 3-5 $\rho_0$ , and the main constituent will be the up and down quarks only. In these region, strangeness is not the main excitation mode. Moreover, HDL correction is not relevant for strange quark as its mass ( $m_s \sim 95$  MeV [38]) is comparable to  $\mu_f$ .

The ideal quark gas contribution (Fig. 1(a)) can be calculated as

$$\ln \mathcal{Z}_{\Omega_{q,0}} \simeq \beta V \left( \frac{N_c}{12} \sum_{f=u,d} \frac{\mu_f^4}{\pi^2} \right), \quad (24)$$

where the absolute divergence known as “vacuum energy” has been neglected. Hereafter, the matter independent divergences will be neglected as we are only interested in the finite difference between thermodynamic potentials.

The ideal gluon gas contribution can be neglected ( $\ln \mathcal{Z}_{\Omega_{g,0}} \simeq (T \sim g\mu)^4$ ). The remaining HDL approximated gluonic ring diagrams should be resummed. We denote the resummation as “the gluon resummation” ( $\ln \mathcal{Z}_{\Omega_{g,\text{HDL}}}$ ), which corresponds to the diagrams Fig. 1(b)-(d):

$$\begin{aligned} \ln \mathcal{Z}_{\Omega_{g,\text{HDL}}} &= -\frac{(N_c^2 - 1)}{2} \beta V \int \frac{d^4 Q}{(2\pi)^4} \ln [1 + \Pi_{\mu\nu}(Q) D_F^{\nu\mu}(Q)] \\ &= -\frac{(N_c^2 - 1)}{2} \beta V \int \frac{d^4 Q}{(2\pi)^4} \left( \ln \left[ 1 + \delta\Pi^L(Q) \frac{1}{Q^2} \right] + 2 \ln \left[ 1 + \delta\Pi^T(Q) \frac{1}{Q^2} \right] \right) \\ &= \mathcal{L} + 2\mathcal{T}, \end{aligned} \quad (25)$$

where  $\mathcal{L}$  and  $\mathcal{T}$  are defined as follows:

$$\mathcal{L} \equiv -\frac{(N_c^2 - 1)}{2} \beta V \int \frac{d^4 Q}{(2\pi)^4} \ln \left[ 1 + m^2 \left( 1 - \frac{i\omega}{2q} \ln \frac{i\omega + q}{i\omega - q} \right) \frac{1}{q^2} \right], \quad (26)$$

$$\mathcal{T} \equiv -\frac{(N_c^2 - 1)}{2} \beta V \int \frac{d^4 Q}{(2\pi)^4} \ln \left[ 1 + \frac{m^2}{2} \left( \frac{i\omega}{q} \right) \left[ \left( 1 - \left( \frac{i\omega}{q} \right)^2 \right) Q_0 \left( \frac{i\omega}{q} \right) + \left( \frac{i\omega}{q} \right) \right] \frac{1}{Q^2} \right]. \quad (27)$$

Next, we take  $T \rightarrow 0$  limit and use the spatial  $d$ -dimension regularization,

$$\int \frac{d^4 Q}{(2\pi)^4} \Rightarrow \int \frac{d\omega}{(2\pi)} \bar{\mu}^{3-d} \int \frac{d^d q}{(2\pi)^3}, \quad (28)$$

where the discrete sum over Matsubara frequencies  $T \sum_n$  changes into a finite continuous integration,  $\int d\omega/(2\pi)$ , in the  $T \rightarrow 0$  limit. One can then regularize the divergence coming from the remaining  $d = D - 1$  spatial dimension with  $g^2 = g_0^2 \bar{\mu}^{2\epsilon} = g_0^2 \bar{\mu}^{3-d}$ . By extracting out the overall mass dimension as  $\bar{\mu}^{3-d}$  in front of the  $d$ -dimension momentum integration, one can set the coupling constant to be dimensionless and define the one-loop counterterm in  $\ln \mathcal{Z}_{\Omega_{g,\text{HDL}}}$ . A similar rigorous regularization scheme in Coulomb gauge has been already reported in Ref. [25]. In the present work, the calculation is performed in the covariant gauge and the result is totally equivalent to the calculation of Ref. [25, 27] at  $T = 0$ .

As both  $\delta\Pi^L(Q)$  and  $\delta\Pi^T(Q)$  depend only on  $i\omega/q$ , the integration variable  $\omega$  can be re-scaled as  $\omega \rightarrow q\bar{\omega}$  to scale out the dimensions of the integration:

$$\mathcal{L} = -\frac{(N_c^2 - 1)}{2} \beta V \frac{1}{(2\pi)} \frac{d\Omega_d}{(2\pi)^d} \bar{\mu}^{3-d} 2 \int_0^\infty d\bar{\omega} \int_0^\infty dq q^d \ln [q^2 + \tilde{\mathcal{L}}(\bar{\omega})], \quad (29)$$

where the absolute divergence proportional to  $\int_0^\infty dq q^d \ln q^2$  has been neglected and  $\tilde{\mathcal{L}}(\bar{\omega})$  is given as

$$\tilde{\mathcal{L}}(\bar{\omega}) \equiv m^2 \left( 1 - \frac{i\bar{\omega}}{2} \ln \frac{i\bar{\omega} + 1}{i\bar{\omega} - 1} \right). \quad (30)$$

Using the following integration formula [25]

$$\int_0^\infty dk k^\alpha \ln(k^2 + m^2) = \frac{\Gamma(\frac{1+\alpha}{2}) \Gamma(\frac{1-\alpha}{2})}{\alpha + 1} m^{\alpha+1} + \text{absolute divergence}, \quad (31)$$

and the  $\overline{\text{MS}}$  subtraction scheme  $\mu_4^2 = (4\pi/e^\gamma)\bar{\mu}^2$ , the integration can be arranged as

$$\begin{aligned} \mathcal{L} &= (N_c^2 - 1)\beta V \frac{1}{(2\pi)} \frac{d\Omega_3}{(2\pi)^3} \frac{(m^2)^2}{4} \left[ \left(1 - \ln \frac{m^2}{\pi\mu_4^2}\right) \alpha - \beta + \frac{1}{\epsilon} \alpha \right] \\ &\Rightarrow \beta V \left[ \alpha_s^2 \frac{2}{\pi} \left( \sum_{f=u,d} \frac{\mu_f^2}{\pi^2} \right)^2 \left[ \left(1 - \ln 2 - \ln \left( \sum_{f=u,d} \frac{\mu_f^2}{\pi^2} \frac{1}{\mu_4^2} \right) - \ln \alpha_s \right) \alpha - \beta \right] \right]_{\text{finite}}, \end{aligned} \quad (32)$$

where  $N_c = 3$  and the constants are calculated as

$$l(\bar{\omega}) \equiv 1 + \bar{\omega} \left( \arctan \bar{\omega} - \frac{\pi}{2} \right), \quad (33)$$

$$\alpha \equiv \int_0^\infty d\bar{\omega} l(\bar{\omega})^2 = \frac{1}{3} \pi (1 - \ln 2) = 0.321336, \quad (34)$$

$$\beta \equiv \int_0^\infty d\bar{\omega} l(\bar{\omega})^2 \ln l(\bar{\omega}) = -0.176945. \quad (35)$$

$\mathcal{T}$  can be obtained in a similar way. After using the same re-scaling  $\omega \rightarrow q\bar{\omega}$ , the integration can be written as

$$\mathcal{T} = -\frac{(N_c^2 - 1)}{2} \beta V \frac{1}{(2\pi)} \frac{d\Omega_d}{(2\pi)^d} \bar{\mu}^{3-d} 2 \int_0^\infty d\bar{\omega} \int_0^\infty dq q^d \ln [q^2 + \tilde{\mathcal{T}}(\bar{\omega})], \quad (36)$$

where  $\tilde{\mathcal{T}}(\bar{\omega})$  is given as

$$\tilde{\mathcal{T}}(\bar{\omega}) \equiv \frac{m^2}{2} \left( -\frac{\bar{\omega}^2}{\bar{\omega}^2 + 1} + \frac{i\bar{\omega}}{2} \ln \frac{i\bar{\omega} + 1}{i\bar{\omega} - 1} \right). \quad (37)$$

Again, using similar method, the integration can be arranged as

$$\begin{aligned} \mathcal{T} &= (N_c^2 - 1)\beta V \frac{1}{(2\pi)} \frac{d\Omega_3}{(2\pi)^3} \frac{(m^2)^2}{8} \left[ \left(1 - \ln \frac{m^2}{2\pi\mu_4^2}\right) \frac{1}{2} \bar{\alpha} - \frac{1}{2} \bar{\beta} + \frac{1}{2\epsilon} \bar{\alpha} \right] \\ &\Rightarrow \beta V \left[ \alpha_s^2 \frac{1}{\pi} \left( \sum_{f=u,d} \frac{\mu_f^2}{\pi^2} \right)^2 \left[ \left(1 - \ln \left( \sum_{f=u,d} \frac{\mu_f^2}{\pi^2} \frac{1}{\mu_4^2} \right) - \ln \alpha_s \right) \frac{1}{2} \bar{\alpha} - \frac{1}{2} \bar{\beta} \right] \right]_{\text{finite}}, \end{aligned} \quad (38)$$

where  $N_c = 3$  and the constants are calculated as

$$t(\bar{\omega}) \equiv -\frac{\bar{\omega}^2}{\bar{\omega}^2 + 1} + \bar{\omega} \left( \frac{\pi}{2} - \arctan \bar{\omega} \right), \quad (39)$$

$$\bar{\alpha} \equiv \int_0^\infty d\bar{\omega} t(\bar{\omega})^2 = \frac{1}{12} \pi (-5 + 8 \ln 2) = 0.142727, \quad (40)$$

$$\bar{\beta} \equiv \int_0^\infty d\bar{\omega} t(\bar{\omega})^2 \ln t(\bar{\omega}) = -0.200869. \quad (41)$$

The overall divergence has been subtracted by the following counterterm ( $\epsilon \rightarrow 0$ )

$$\Delta \ln \mathcal{Z}_\Omega = -(3^2 - 1) \beta V \frac{1}{(2\pi)} \frac{d\Omega_3}{(2\pi)^3} \frac{(m^2)^2}{4} \left( \alpha + \frac{1}{2} \bar{\alpha} \right) \frac{1}{\epsilon}. \quad (42)$$

For the quarks, the resummation can be done in a similar way as in the gluon case [27]. We adopt the result from Ref. [27] and denote it as “the quark resummation” ( $\ln \mathcal{Z}_{\Omega_q, \text{HDL}}$ ), which corresponds to the diagrams in Fig. 2:

$$\ln \mathcal{Z}_{\Omega_q, \text{HDL}} = \beta V \frac{1}{4} \sum_{f=u,d} \frac{\mu_f^4}{\pi^2} \left[ -4 \left( \frac{\alpha_s}{\pi} \right) + \left( \frac{8}{3} - \frac{4}{9} \pi^2 \right) \left( \frac{\alpha_s}{\pi} \right)^2 \right]. \quad (43)$$

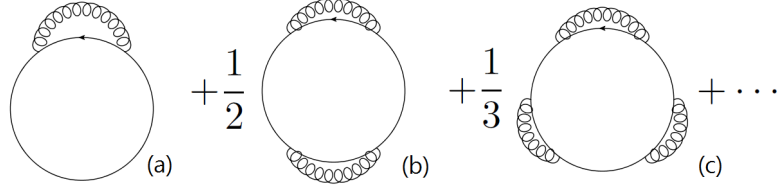


FIG. 2. Diagrammatic description for  $\ln \mathcal{Z}_{\Omega_{q,\text{HDL}}}$ . Each diagram is presented with its symmetric factor.

Finally,  $\ln \mathcal{Z}_\Omega = \ln \mathcal{Z}_{\Omega_{q,0}} + \ln \mathcal{Z}_{\Omega_{g,\text{HDL}}} + \ln \mathcal{Z}_{\Omega_{q,\text{HDL}}}$  can be written as an expansion in  $\alpha_s^n (\ln \alpha_s)^m$  as follows:

$$\begin{aligned} \ln \mathcal{Z}_\Omega = & \beta V \left( \frac{1}{4} \sum_{f=u,d} \frac{\mu_f^4}{\pi^2} \left[ 1 - 4 \left( \frac{\alpha_s}{\pi} \right) + \left( \frac{8}{3} - \frac{4}{9} \pi^2 \right) \left( \frac{\alpha_s}{\pi} \right)^2 \right] \right. \\ & \left. + \alpha_s^2 \frac{2}{\pi} \left( \sum_{f=u,d} \frac{\mu_f^2}{\pi^2} \right)^2 \left[ \left( 1 - \ln \alpha_s - \ln \left( \sum_{f=u,d} \frac{\mu_f^2}{\pi^2} \frac{1}{\mu_4^2} \right) \right) \Lambda_1 - \Lambda_2 - \alpha \ln 2 \right] \right), \end{aligned} \quad (44)$$

where the constants  $\Lambda_1 \equiv \alpha + \frac{1}{2} \bar{\alpha}$  and  $\Lambda_2 \equiv \beta + \frac{1}{2} \bar{\beta}$ . The thermodynamic quantities can be obtained from  $\ln \mathcal{Z}_\Omega$ :

$$\begin{aligned} \frac{\Omega(\mu)}{V} = & \frac{\langle \hat{H} \rangle - \vec{\mu} \cdot \langle \vec{\hat{N}} \rangle}{V} = -\frac{1}{\beta V} \ln \mathcal{Z}_\Omega \\ = & -\frac{1}{4} \sum_{f=u,d} \frac{\mu_f^4}{\pi^2} \left[ 1 - 4 \left( \frac{\alpha_s}{\pi} \right) + \left( \frac{8}{3} - \frac{4}{9} \pi^2 \right) \left( \frac{\alpha_s}{\pi} \right)^2 \right] \\ & - \alpha_s^2 \frac{2}{\pi} \left( \sum_{f=u,d} \frac{\mu_f^2}{\pi^2} \right)^2 \left[ \left( 1 - \ln \alpha_s - \ln \left( \sum_{f=u,d} \frac{\mu_f^2}{\pi^2} \frac{1}{\mu_4^2} \right) \right) \Lambda_1 - \Lambda_2 - \alpha \ln 2 \right], \end{aligned} \quad (45)$$

$$\begin{aligned} \rho_i(\mu) = & \frac{\langle \hat{N}_i \rangle}{V} = \frac{1}{\beta V} \frac{\partial}{\partial \mu_i} \ln \mathcal{Z}_\Omega, \\ = & \frac{\mu_i^3}{\pi^2} \left[ 1 - 4 \left( \frac{\alpha_s}{\pi} \right) + \left( \frac{8}{3} - \frac{4}{9} \pi^2 \right) \left( \frac{\alpha_s}{\pi} \right)^2 \right] \\ & + \alpha_s^2 \frac{8}{\pi} \left( \sum_{f=u,d} \frac{\mu_f^2}{\pi^2} \right) \frac{\mu_i}{\pi^2} \left[ \left( \frac{1}{2} - \ln \alpha_s - \ln \left( \sum_{f=u,d} \frac{\mu_f^2}{\pi^2} \frac{1}{\mu_4^2} \right) \right) \Lambda_1 - \Lambda_2 - \alpha \ln 2 \right], \end{aligned} \quad (46)$$

$$\begin{aligned} \epsilon(\mu) = & \frac{\langle \hat{H} \rangle}{V} = -\frac{1}{V} \left( \frac{\partial}{\partial \beta} - \frac{1}{\beta} \vec{\mu} \cdot \frac{\partial}{\partial \vec{\mu}} \right) \ln \mathcal{Z}_\Omega \\ = & \frac{3}{4} \sum_{f=u,d} \frac{\mu_f^4}{\pi^2} \left[ 1 - 4 \left( \frac{\alpha_s}{\pi} \right) + \left( \frac{8}{3} - \frac{4}{9} \pi^2 \right) \left( \frac{\alpha_s}{\pi} \right)^2 \right] \\ & + \alpha_s^2 \frac{6}{\pi} \left( \sum_{f=u,d} \frac{\mu_f^2}{\pi^2} \right)^2 \left[ \left( \frac{1}{3} - \ln \alpha_s - \ln \left( \sum_{f=u,d} \frac{\mu_f^2}{\pi^2} \frac{1}{\mu_4^2} \right) \right) \Lambda_1 - \Lambda_2 - \alpha \ln 2 \right], \end{aligned} \quad (47)$$

where  $\alpha_s(\mu_4)$  is assigned for 2-loop renormalization in  $\overline{\text{MS}}$  scheme:

$$\begin{aligned} \alpha_s(\mu_4) = & \frac{4\pi}{\beta_0 \ln(\mu_4^2/\Lambda_{\overline{\text{MS}}}^2)} \left( 1 - \frac{2\beta_1 \ln(\ln(\mu_4^2/\Lambda_{\overline{\text{MS}}}^2))}{\beta_0^2 \ln(\mu_4^2/\Lambda_{\overline{\text{MS}}}^2)} \right), \\ \beta_0 = & (11N_c - 2N_f)/3, \\ \beta_1 = & (34N_c^2 - 13N_cN_f + 3N_f/N_c)/6, \end{aligned} \quad (48)$$

where  $\mu_4$  will be parameterized as  $1.5\mu \leq \mu_4 \leq 4\mu$ .  $\mu$  is the quark chemical potential at iso-spin symmetric condition. As discussed in Ref. [26, 27, 39], if higher order calculation can be obtained,  $\alpha_s$  can be determined by solving the



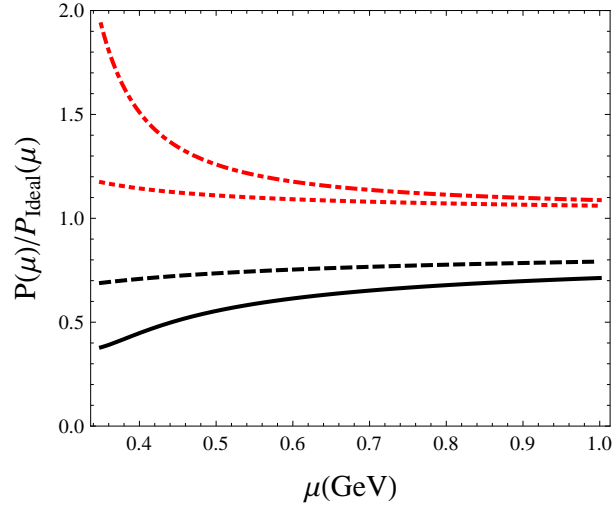


FIG. 3. (Color online) The ratio  $P(\mu)/P_{\text{ideal}}(\mu)$  with  $\mu_4 = 2\mu$  for red dot-dashed and black solid line, and  $\mu_4 = 4\mu$  for red dotted and black dashed line. The upper two lines are obtained with gluon resummation results and the lower two lines from adding the quark resummation to the gluon resummation.

self-consistent gap equation for the thermodynamic potential. On the other hand, since we have calculated only to leading order in the HDL perturbation, the effect of  $\alpha_s(\mu_4)$  is investigated by varying  $1.5\mu \leq \mu_4 \leq 4\mu$ .

Fig.3 shows the ratio  $P(\mu)/P_{\text{ideal}}(\mu)$  with various  $\mu_4$ . The upper two lines are obtained with contributions coming from gluon HDL resummation results and the lower two lines from adding the quark resummation to the gluon HDL resummation. Red dotdashed and black solid line are obtained with  $\mu_4 = 2\mu$ . Red dotted and black dashed line are obtained with  $\mu_4 = 4\mu$ . One can find that HDL resummed  $P(\mu)/P_{\text{ideal}}(\mu)$  varies in a large band in Fig 3. As can be expected,  $P(\mu)/P_{\text{ideal}}(\mu)$  approaches to 1 as  $\mu_4$  becomes larger. Here one notes that the gluon resummation gives the enhanced ratio ( $P(\mu)/P_{\text{ideal}}(\mu) \sim 1.1$  at  $\mu = 1$  GeV), while inclusion of the quark resummation gives the reduced ratio ( $P(\mu)/P_{\text{ideal}}(\mu) \sim 0.8$  at  $\mu = 1$  GeV). The reduction of pressure due to the inclusion of quark contributions is consistent with previous studies that include a massive strangeness and the quark interaction part up to  $O(\alpha_s^2)$  [52, 53]. Based on the result of Ref. [52], the authors in Ref. [53] also find a similar behavior of the pQCD ratio ( $0.6 < P(\mu)/P_{\text{ideal}}(\mu) < 0.8$  near  $\mu = 1$  GeV) as shown in Fig. 2 of their work. While the quark resummation enhances the  $O(\alpha_s)$  correction compared to the weak coupling expansion [26, 27], introducing the quark phase in the core of a neutron star in general allows for a more massive neutron star that is consistent with recent measurement as was discussed in Ref. [53]. To fix the scale, we plotted the quark energy density in Fig. 4. One notes that the value becomes negative at lower  $\mu$  when  $\mu_4 = 1.5\mu$ . This behavior is due to the increase in  $\alpha_s$  at small scale, which is a consequence of asymptotic freedom. To make sure the quark energy density be positive, we choose  $\mu_4 = 2.5\mu$  throughout this section.

From Eqs. (45)-(47), one can find that the thermodynamic quantities depend mainly on quark and gluon rest mass  $m^2, m_f^2 \sim g^2\mu^2/\pi^2$ . In the HDL resummation, the gluonic contribution is dominated by the longitudinal gluon rest mass. Iso-spin asymmetric properties can be accounted for by assigning asymmetric quark chemical potential to the quark and gluon rest mass.

### C. Cold dense matter symmetry energy at normal phase

In the iso-spin asymmetric matter, the light quark chemical potential can be defined as follows:

$$\mu_d^u = \mu \left( 1 \mp \frac{1}{3} I_B \right)^{\frac{1}{3}}. \quad (49)$$

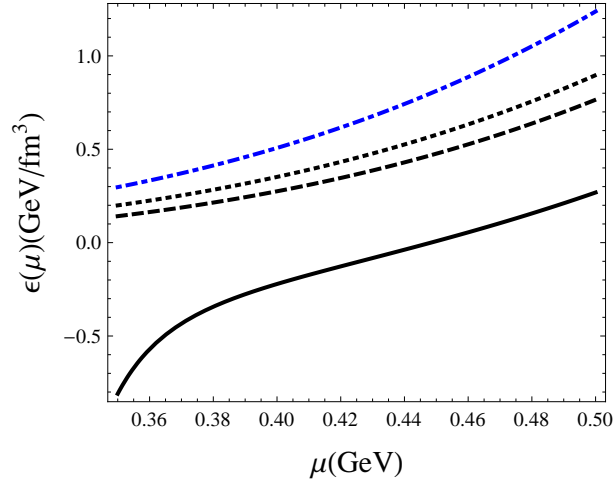


FIG. 4. (Color online) Quark energy density  $\epsilon(\mu)$  with various  $\mu_4$ . Blue dot-dashed line is for ideal quark gas. Black lines represent the result for quark and gluon resummation. Black dotted line represents the result with  $\mu_4 = 4\mu$ , black dashed line with  $\mu_4 = 2.5\mu$  and black solid line with  $\mu_4 = 1.5\mu$ , respectively.

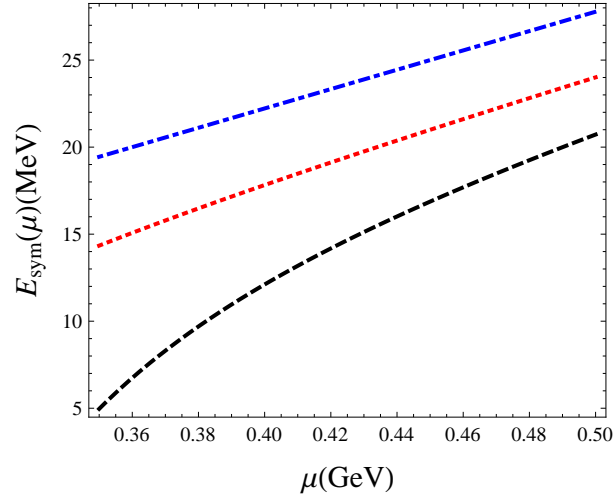


FIG. 5. (Color online) Quark matter symmetry energy at normal phase ( $\mu_4 = 2.5\mu$ ). Blue dot-dashed line represents symmetry energy of ideal quark gas. Red dotted line represents symmetry energy with gluon resummation only while black dashed line represents the result after adding the quark resummation.

By assigning this relation to Eqs. (46)-(47), quark matter symmetry energy in the normal phase can be obtained from energy per baryon number as follows:

$$\frac{\epsilon(\mu, I_B)}{\rho_B(\mu, I_B)} = \bar{E}(\mu, I_B) = E_0(\mu, I_B) + \bar{E}_{sym}(\mu)I_B^2 + O(I_B^4) + \dots, \quad (50)$$

$$E_{sym}(\mu) = \frac{1}{2!} \frac{\partial^2}{\partial I_B^2} \bar{E}(\mu, I_B), \quad (51)$$

where  $\rho_B = (\rho_u + \rho_d)/3$ ,  $I_B = 3(\rho_u - \rho_d)/(\rho_u + \rho_d)$ .

The quark matter symmetry energy is plotted in Fig. 5. Here one can check that the symmetry energy is reduced by HDL resummation compared to the ideal quark gas case. It reduces slightly when the gluon HDL resummation is included and more when the quark resummation is added. Hence, one can conclude that HDL resummed interaction makes it easier for the cold quark matter to be iso-spin asymmetric: it costs less energy to reach the same iso-spin asymmetry in comparison with the ideal quark gas. When temperature becomes even lower, additional non-perturbative effect should be considered. We discuss an effect in the next section.

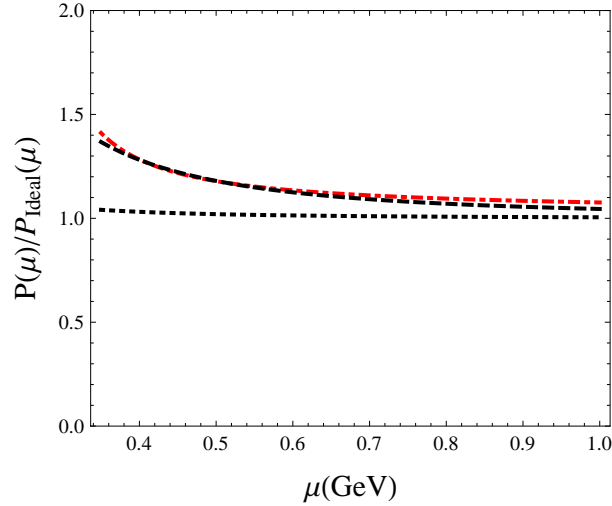


FIG. 6. (Color online) Ratio  $P(\mu)/P_{\text{ideal}}(\mu)$  with various  $\Delta$ . Black dashed line represents  $P(\mu)/P_{\text{ideal}}(\mu)$  with  $\Delta = 150$  MeV. Black dotted line represents  $P(\mu)/P_{\text{ideal}}(\mu)$  with  $\Delta = 50$  MeV. Red dot-dashed line represents  $P(\mu)/P_{\text{ideal}}(\mu)$  of the gluon resummation with  $\mu_4 = 2.5\mu$  at normal phase as a criterion.

### III. SYMMETRY ENERGY AT EXTREMELY LOW TEMPERATURE

Now we consider the extremely cold matter case ( $T \ll g\mu$ ). As we have found in the previous section, the gluonic contribution to symmetry energy depends mainly on the iso-spin asymmetric quark and gluon rest masses which comes from HDL resummation. In this section we will concentrate on the iso-spin asymmetric corrections which come from extremely cold matter.

In such a cold condition, for ideal quark gas, all fermion will be confined in their Fermi sea without large fluctuation. When interaction turns on, one can consider nontrivial correlation between fermions and related effect. According to effective descriptions based on Wilson's renormalization group approach [14–17], when one scales the momentum to near the Fermi surface, the interaction between fermions with opposite momenta becomes marginal. So if the interaction is attractive, it is natural to form a bosonic condensate for two fermions on the same Fermi sea with opposite Fermi velocity (BCS pairing) [18]. In QCD, one-gluon exchange via color antisymmetric channel naturally provides attractive interaction. So in the extremely low temperature limit, one should consider the effects of color superconductivity [19, 20].

#### A. 2-color superconductivity (2SC) and symmetry energy

First, we confine the density region to be similar to that of previous section:  $3-5\rho_0$ . The corresponding quark chemical potential then ranges from  $0.35 \text{ GeV} \leq \mu \leq 0.5 \text{ GeV}$ . In this region, there is a mismatch of Fermi momentum between light quark and strange quark ( $p_F^u \simeq p_F^d \simeq \mu \gg p_F^s = \sqrt{\mu_s^2 - m_s^2}$ ) even if same numbers of light quarks and strange quarks exist. Thus when strangeness is not the main excitation, only light quark flavor can participate in formation of BCS condensate (2SC) [24]. So again, we consider only light quark flavors in this section. The spin-0 channel forms the most stable diquark condensate as quark pairs from the whole iso-tropic Fermi surface can contribute to the pairing. This type of condensation requires the quark pair to be on the same Fermi surface but with opposite velocity and spin alignment, such that they are in same helicity state. As color configuration is already antisymmetric, to obtain total antisymmetric wave function, quark flavors should be in the antisymmetric configuration. The diquark condensate can then be summarized as follows [32, 33]:

$$\langle \psi_{L,\alpha i}^T C \psi_{L,\beta j} \rangle = -\langle \psi_{R,\alpha i}^T C \psi_{R,\beta j} \rangle = \frac{\Delta}{2} \epsilon_{\alpha\beta 3} \epsilon_{ij 3}, \quad (52)$$

where  $C = i\sigma_2$ ,  $\Delta$  is the superconducting gap size, the Greek index represents color, the Latin index represents quark flavor and  $L, R$  represents chirality of the quarks. The condensate can be included in the interaction terms of

Lagrangian as the invariant coupling [32, 33]:

$$\mathcal{L}_\Delta = -\frac{\Delta}{2}\psi_L^T C\epsilon\psi_L\epsilon - (L \rightarrow R) + \text{h.c.}, \quad (53)$$

where  $\epsilon = i\sigma_2$  on SU(2) color-flavor index.

Although the existence of the 2SC phase is based on the iso-spin symmetric Fermi sea of the light quarks at zero temperature, the phase can persist up to some critical temperature and iso-spin asymmetry determined by the Clogston limit  $\delta\mu = \Delta/\sqrt{2}$  [20, 49]. Specifically, the PNJL model calculation [50] suggests that the critical temperature is in the order of 50 MeV. Moreover, even if the pure neutron matter ( $I_B = 1$ ) changes into quark matter, the difference between Fermi sea of  $u$  and  $d$  quark will be  $\delta\mu \sim 0.23\mu$ . Hence, assuming that the gap also grows with the chemical potential, the Clogston limit can be satisfied at higher density. In realistic heavy ion collisions involving ordinary dense matter ( $I_B < 1$ ) the iso-spin difference will be smaller ( $\delta\mu < 0.23\mu$ ) and the possibility to probe 2SC matter remains. Such a matter can be produced in the planned FAIR experiment [51].

If one considers only the ideal quark contribution and the correction coming from the non-perturbative gap contribution, the free energy can be written as follows [28, 40]:

$$\begin{aligned} \Omega_\Delta(\mu) &= \Omega_{q,0}(\mu) + \delta\Omega_{q,\Delta}(\mu) \\ &= -\frac{1}{12} \sum_{f=u,d}^{N_c} \frac{\mu_f^4}{\pi^2} + \sum_i^{2\text{SC}} \frac{\mu_i^2}{\pi^2} \int dp_4 \left[ \sqrt{p_4^2 + \Delta(p_4)^2} - p_4 - \frac{\Delta(p_4)^2}{2\sqrt{p_4^2 + \Delta(p_4)^2}} \right] \\ &\simeq -\frac{1}{12} \sum_{f=u,d}^{N_c} \frac{\mu_f^4}{\pi^2} - \sum_i^{2\text{SC}} \frac{\mu_i^2 \Delta^2}{4\pi^2}, \end{aligned} \quad (54)$$

where the approximation in the last line follows from Ref. [20, 29]. In the 2SC phase, the 2-color and 2-flavor states participate in the condensation and hence share the same chemical potential. Here, we ignore  $u$  and  $d$  quark chemical potential asymmetry coming from the difference in their charge. Then, the correction from 2SC can be written as  $\delta\Omega_{q,\Delta}(\mu) = -4(\mu^2 \Delta^2 / 4\pi^2)$  without the iso-spin index.

The gap size  $\Delta$  has been determined in several studies. The first work in non-perturbative regime was reported by D. T. Son as  $\Delta/\mu = (b/g^5)\exp(-3\pi^2/\sqrt{2}g)$  by solving the self-consistent gap equation in using HDL approximation [21]. Schäfer and Wilczek [22] determined the constant to be  $b = 256\pi^4$  and the corresponding gap size in  $\mu = 400$  MeV to be in the order of 100 MeV. The estimation from HDET [33] was found to be in the order of 50 MeV, while model calculation [23] gives the value in the order of 100 MeV.

In Fig. 6 we plotted  $P(\mu)/P_{\text{ideal}}(\mu)$  in the range of  $50 \text{ MeV} \leq \Delta \leq 200 \text{ MeV}$ . As one can expect, as  $\Delta$  becomes larger, the pressure deviates more from the ideal quark gas limit. When  $\Delta = 150$  MeV,  $P(\mu)/P_{\text{ideal}}(\mu)$  shows similar behavior to  $P(\mu)/P_{\text{ideal}}(\mu)$  at normal phase (with only the gluon resummation,  $\mu_4 = 2.5\mu$ ). Also with the gap size  $\Delta = 150$  MeV, large enough Clogston limit ( $\delta\mu \sim 0.23\mu < \Delta/\sqrt{2}$ ) can be obtained in  $3\text{--}5\rho_0$  region. For gluonic terms, at 2SC phase, HDL effective Lagrangian can not be inserted directly to the present Lagrangian as the quasi-loop correction in 2SC phase is different with the HDL of normal phase. However, it was argued in Ref. [41] that the coupling in such cases will be weak, hence, may give minimal effect to the free energy. In this section, we choose the  $\Delta = 150$  MeV to match the value of the free energy calculated in both phases as shown in the upper two lines in Fig. 6.

The symmetry energy in 2SC phase can be obtained in similar way from the energy per quasi-baryon number:

$$\frac{\epsilon(\mu, I_{\bar{B}})}{\rho_{\bar{B}}(\mu, I_{\bar{B}})} = \bar{E}(\mu, I_{\bar{B}}) = E_0(\mu, I_{\bar{B}}) + \bar{E}_{\text{sym}}(\mu) I_{\bar{B}}^2 + O(I_{\bar{B}}^4) + \dots, \quad (55)$$

$$E_{\text{sym}}^{2\text{SC}}(\mu) = \frac{1}{2!} \frac{\partial^2}{\partial I_{\bar{B}}^2} \bar{E}(\mu, I_{\bar{B}}), \quad (56)$$

where  $\rho_{\bar{B}} = (\rho_{\text{unpaired}} + \rho_{\text{paired}})/3$  and  $I_{\bar{B}} = I_B/3$  as asymmetrizable unpaired states reduce to 1/3 of the normal

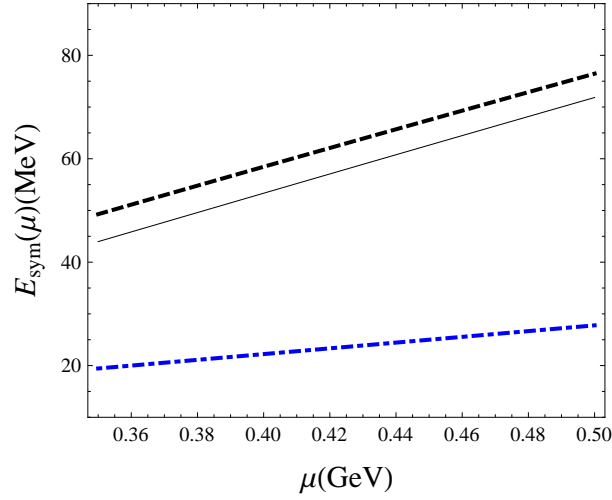


FIG. 7. (Color online) Quark matter symmetry energy at 2SC phase. Blue dot-dashed line represents symmetry energy of the ideal quark gas. Black solid line represents symmetry energy at 2SC phase with  $\Delta = 200$  MeV. Black dashed line represents symmetry energy at 2SC phase with  $\Delta = 150$  MeV.

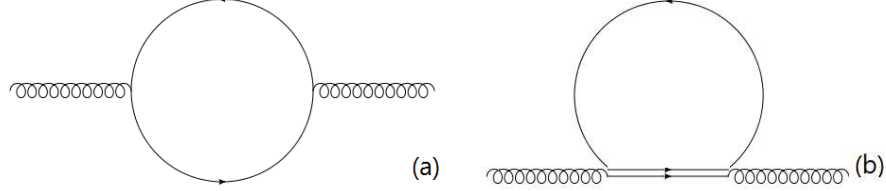


FIG. 8. Relevant one loop diagrams for the gluon rest masses. (a) represents the ordinary polarization part. (b) represents the momentum independent tadpole part which gives counterterm to transverse gluon mass. Double line in (b) represents propagation in Dirac sea mode [30, 31].

phase. From  $\Omega_\Delta(\mu)$ , the quasi-quark number density and the energy density can be obtained as follows:

$$\rho_i(\mu) = \frac{1}{3} \frac{\mu_i^3}{\pi^2}, \quad (57)$$

$$\rho_{\Delta i}(\mu) = \frac{1}{3} \frac{\mu_i^3}{\pi^2} + \frac{\mu_i \Delta^2}{2\pi^2}, \quad (58)$$

$$\begin{aligned} \epsilon_\Delta(\mu) &= \epsilon_{\text{unpaired}}(\mu) + \epsilon_{\text{paired}}(\mu) \\ &= \frac{1}{4} \sum_i^{\text{unpaired}} \frac{\mu_i^4}{\pi^2} + \frac{1}{4} \sum_i^{2\text{SC}} \left[ \frac{\mu_i^4}{\pi^2} + \frac{\mu_i^2 \Delta^2}{\pi^2} \right], \end{aligned} \quad (59)$$

where  $\mu_d^u = \mu(1 \mp I_{\tilde{B}})^{\frac{1}{3}}$  is assigned only for the unpaired states, while the other quasi-quark states share symmetric chemical potential  $\mu$ . The symmetry energy in 2SC phase is plotted in Fig. 7. It is substantially larger than the symmetry energy in the normal phase: it costs more energy to reach the same iso-spin asymmetry in comparison with the normal phase. This large difference may lead to quite different phenomenological prediction, which will be discussed in last section. As the gluonic coupling will be weak [41], the result in Fig. 7 could be dominant contribution.

### B. Gluon rest masses in iso-spin asymmetric 2SC matter

While we expect the gluonic contribution to the symmetry energy in the 2SC phase to be weak, it is worth while to estimate it. As we have found in the previous section, contribution of HDL resummation to the symmetry energy in the normal phase is determined dominantly by the quark and gluon rest masses. Among them, the gluon resummation with hard quark loop correction seems suitable for physically relevant condition. In the 2SC phase, the gluonic

	$a, b = 1, 2, 3$	$a, b = 4, 5, 6, 7$	$a, b = 8$
Paired (A=0)	$\xi_{AA}^{ab} = \frac{1}{4}\delta^{ab}$	$\xi_{AA}^{ab} = \frac{1}{8}\delta^{ab}$	$\xi_{AA}^{ab} = \frac{1}{12}\delta^{ab}$
Paired (A=1,2,3)	$\xi_{AA}^{ab} = \frac{1}{4}\delta^{ab}$	$\xi_{AA}^{ab} = \frac{1}{8}\delta^{ab}$	$\xi_{AA}^{ab} = \frac{1}{12}\delta^{ab}$
Unpaired (A=4,5)	$\xi_{AA}^{ab} = 0$	$\xi_{AA}^{ab} = \frac{1}{4}\delta^{ab}$	$\xi_{AA}^{ab} = \frac{1}{3}\delta^{ab}$

TABLE I. List of  $\xi_{AA}^{ab}$ . Small letters a,b represent adjoint color index of the external gluon. Capital letter A represents the quasi-quark state in the loop of the tadpole diagram (Fig. 8(b)).

excitation will be soft or super-soft ( $Q \sim T \ll g\mu$ ) and we assume that the qualitative behavior of the symmetry energy in 2SC phase also depends mainly on the quasi-quark loop correction in small momentum limit. By using HDET [30–33], the rest masses of gluon can be simply calculated. In HDET, matter part of Lagrangian can be written as follows [30, 31]:

$$\begin{aligned} \mathcal{L}_q &= \sum_f \bar{\psi}_f (i\not{D} + \mu_f \gamma_0) \psi_f \\ &= \sum_{\vec{v}_f} \sum_f \left( \psi_{f+}^\dagger (iV \cdot D) \psi_{f+} - \psi_{f+}^\dagger \frac{1}{2\mu_f + i\bar{V} \cdot D} (\not{D}_\perp)^2 \psi_{f+} \right), \end{aligned} \quad (60)$$

where  $V^\mu = (1, \vec{v}_f)$  and  $\bar{V}^\mu = (1, -\vec{v}_f)$  with Fermi velocity  $\vec{v}_f$ ,  $\not{D}_\perp = \gamma_\perp^\mu D_\mu$  with  $\gamma_\perp^\mu = \gamma^\mu - \gamma_\parallel^\mu$  and  $\gamma_\parallel^\mu = (\gamma^0, \vec{v}_f \vec{v}_f \cdot \vec{\gamma})$  and  $\psi_{f+}$  represents positive energy projection of quark field. The irrelevant quark mode  $\psi_{f-}$  is integrated out by using equation of motion:  $\psi_{f-} = -i\gamma^0/(2\mu_f + i\bar{V} \cdot D) \not{D}_\perp \psi_{f+}$ .

In 2SC phase, the quasi-quark states can be represented as follows [32]:

$$\begin{aligned} \psi_{\pm, \alpha i} &= \sum_{A=0}^5 \frac{(\tilde{\lambda}_A)_{\alpha i}}{\sqrt{2}} \psi_\pm^A, \quad \chi^A = \begin{pmatrix} \psi_+^A \\ C\psi_-^{A*} \end{pmatrix}, \\ \tilde{\lambda}_0 &= \frac{1}{\sqrt{3}}\lambda_8 + \frac{2}{3}I; \quad \tilde{\lambda}_A = \lambda_A \quad (A = 1, 2, 3); \quad \tilde{\lambda}_4 = \frac{1}{\sqrt{2}}(\lambda_4 - i\lambda_5); \quad \tilde{\lambda}_5 = \frac{1}{\sqrt{2}}(\lambda_6 - i\lambda_7), \end{aligned} \quad (61)$$

where  $A = 0, 1, 2, 3$  denotes gapped states,  $A = 4, 5$  denotes ungapped states and  $\lambda_i$  is Gell-mann matrix. Here, ‘ $\pm$ ’ do not represent the energy eigenstate. It represents direction of Fermi velocity. Incorporating this representation and the gap interaction term (53), the Lagrangian can be written in Nambu-Gorkov form [32, 33] as

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \sum_{\vec{v}_f} \sum_{A,B=0}^{\text{half}} \left[ \chi^{A\dagger} \begin{pmatrix} iV \cdot \partial \delta_{AB} & \Delta_{AB} \\ \Delta_{AB} & i\bar{V} \cdot \partial \delta_{AB} \end{pmatrix} \chi^B + ig A_\mu^a \chi^{A\dagger} \begin{pmatrix} iV^\mu \kappa_{AaB} & 0 \\ 0 & -i\bar{V}^\mu \kappa_{AaB}^* \end{pmatrix} \chi^B \right. \\ &\quad \left. + g^2 A_\mu^c A_\nu^d \chi^{A\dagger} \begin{pmatrix} \frac{1}{2\mu_f + i\bar{V} \cdot D} \xi_{AB}^{cd} & 0 \\ 0 & \frac{1}{2\mu_f + iV \cdot D^*} \xi_{AB}^{cd*} \end{pmatrix} P^{\mu\nu} \chi^B \right] + (L \rightarrow R), \end{aligned} \quad (62)$$

where  $P^{\mu\nu} = g^{\mu\nu} - \frac{1}{2}(V^\mu \bar{V}^\nu + V^\nu \bar{V}^\mu)$  and the constants are defined as

$$\begin{aligned} \kappa_{AaB} &= \frac{1}{2} \text{Tr}[\tilde{\lambda}_A \tau_a \tilde{\lambda}_B], \quad \xi_{AB}^{cd} = \frac{1}{2} \text{Tr}[\tilde{\lambda}_A \tau_c \tau_d \tilde{\lambda}_B], \\ \Delta_{AB} &= \frac{\Delta}{2} \text{Tr}[\epsilon \sigma_A^T \epsilon \sigma_B] \quad (A, B = 0, 1, 2, 3, \text{ otherwise } \Delta_{AB} = 0), \\ &= \Delta_A \delta_{AB} \quad \text{with } \Delta_A = (-\Delta, \Delta, \Delta, \Delta, 0, 0), \end{aligned} \quad (63)$$

where  $\tau_a$  is the fundamental representation of SU(3) generator.

For gluon self energy, the diagrams in Fig. 8 are relevant. The diagram Fig. 8(a) can be calculated from ordinary current-current correlator

$$\Pi_{\mu\nu}^{ab}(q) = g^2 \int d^4x e^{-iq \cdot x} \sum_{\vec{v}_f}^{\text{half}} \langle \Omega | T[J_\mu^a(x, \vec{v}_f) J_\nu^b(0, \vec{v}_f)] | \Omega \rangle, \quad (64)$$

	$a, b = 1, 2, 3$	$a, b = 4, 5, 6, 7$	$a, b = 8$
$\Pi_{00}^{ab}(0)$	0	$\frac{1}{2}m^2\delta^{ab}$	$m^2\delta^{ab}$
$-\Pi_{ij}^{ab}(0)$	0	$\frac{1}{12}g^2\sum_f^{4,5}(\mu_f^2/\pi^2)\delta_{ij}\delta^{ab}$	$\frac{1}{9}[-m^2 + g^2\sum_f^{4,5}(\mu_f^2/\pi^2)]\delta_{ij}\delta^{ab}$

TABLE II. Debye and Meissner masses in the asymmetric 2SC matter. Small letters a,b represent adjoint color index of the external gluon.  $m^2 = (g^2\mu^2/\pi^2)$ .

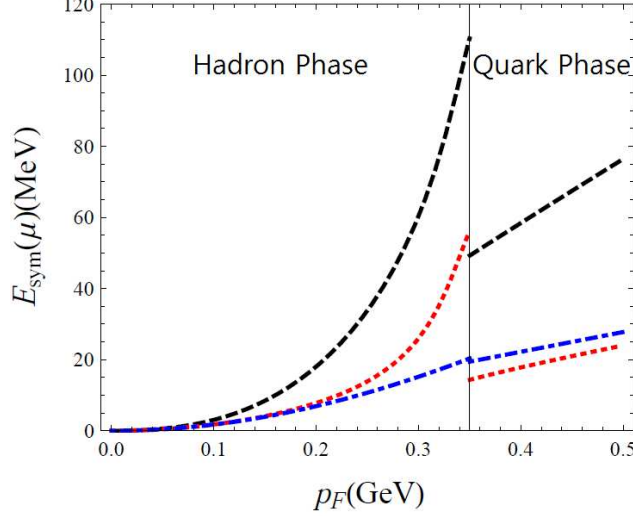


FIG. 9. (Color online) Symmetry energy as a function of the Fermi momentum. Left side: symmetry energy in the hadron phase calculated by using QCD sum rule [46]. Blue dot-dashed line represents the kinetic part of symmetry energy of the ideal nucleon gas. Red dotted line represents the kinetic part of interacting matter. Black dashed line represents the total symmetry energy (kinetic + potential). Right side: quark matter symmetry energy in the normal and 2SC phase. Blue dot-dashed line represents symmetry energy of ideal quark gas. Red dotted line represents symmetry energy in the normal phase with HDL resummation. Black dashed line represents symmetry energy in the 2SC phase. The phase boundary has been set artificially.

where  $|\Omega\rangle$  denotes 2SC ground state and  $J_a^\mu(x, \vec{v}_f)$  can be obtained from the second term in the matter Lagrangian. The diagram Fig. 8(b) directly comes from the last term in the matter Lagrangian which has explicit iso-spin dependence. In iso-spin symmetric condition, with counterterm  $\delta\Pi_{ij}^{ab}(0) = -\frac{1}{3}m^2\delta_{ij}\delta^{ab}$  which should come from Fig. 8(b) type diagrams as it is transverse component, the gluon rest masses can be matched with Debye and Meissner masses calculated in Ref. [36]. In iso-spin asymmetric condition, only through this counterterm, iso-spin asymmetric part of the rest masses can be accounted.

The iso-spin asymmetric part in the whole counterterm  $\delta\Pi_{ij}^{ab}(0)$  can be determined by calculating  $\xi_{AA}^{cd}$ . With  $\xi_{AA}^{ab}$  in Table I, the Debye and Meissner masses in the asymmetric 2SC matter can be obtained. The Debye and Meissner masses are listed in Table II. From Table II, one can find that the rest masses of the gluon in adjoint color  $a = 1, 2, 3$  vanishes. As this type of gluon carries the unbroken SU(2) color charges which participate in the 2SC condensation, the rest masses vanish trivially. The gluon in adjoint color  $a = 4, 5, 6, 7, 8$  has rest masses. It can be noted that only the Meissner mass is iso-spin weakly dependent. For the static quantity such as thermodynamic potential, the contribution of the Debye mass is most important. Consequently, in the 2SC phase, the contribution of the Meissner mass to the symmetry energy will be minimal. However, for dynamical quantities, the situation can be different. The spatial transverse mode of super-soft gluon can acquire nontrivial loop corrections that can be as large as HDL [42–45]. This means that dynamical process which accompanies super-soft gluon in 2SC phase may strongly depend on iso-spin asymmetry. Thus such a process may critically influence observables that can discriminate the existence of 2SC phase and identify the symmetry energy properties of the cold dense matter.

#### IV. DISCUSSION AND CONCLUSIONS

In this work, we calculated the thermodynamic potential and the symmetry energy at the cold dense limit. For the normal phase at low but non-negligible temperature, we first calculated the free energy by using HDL resummation,



which corresponds to calculating quark and gluon ring diagrams with HDL self energy. From a theoretical point of view, the gluon resummation seems to be more suitable for the physical low energy excitations in the cold dense matter. For the quark resummation, we optionally considered the effects of the resummation. All the thermodynamic properties, calculated from the thermodynamic potential, strongly depends on the quark and gluon rest mass. In the gluon self energy as well as the free energy itself, the largest contribution comes from the Debye mass of the longitudinal mode, which comes mainly from the quark-hole loop in HDL self energy, whereas the contribution from the transverse mode, which comes from quark anti-quark loop, is comparatively unimportant. The symmetry energy also comes mainly from the iso-spin dependence of the Debye mass. HDL resummed gluonic interaction gives reducing effect to the symmetry energy.

For the 2SC phase, which can appear when temperature becomes extremely low, the free energy containing non-perturbative condensation should be considered. Following previous studies [28, 29], the free energy can be written in a simplified form as given in Eq. (54), from which one can extract the symmetry energy in the 2SC phase by reducing the formula to have the iso-spin asymmetric factor  $I_{\bar{B}}$ . The symmetry energy becomes almost 3 times that in the normal phase, because 2-color and 2-flavor states are locked in their common Fermi sea, reducing the number of available quarks that can contribute to the asymmetry. Although the symmetry energy becomes larger than the typical value in the normal quark phase, it still remains smaller compared to the expected value in the hadronic phase extrapolated to the phase boundary (Fig. 9). We also estimated the gluonic contribution to the symmetry energy in the 2SC phase. By using HDET [30–33], we found that only the Meissner mass in the 2SC phase has a small iso-spin dependence (Table. II). In another words, as far as the gluons are concerned, the Debye mass is important for the static quantities, and Meissner mass contributes to the symmetry energy, but with an even smaller reducing effect than in the normal phase. Hence, one can just consider the effect of BCS condensation only.

With the results on the cold dense matter symmetry energy in hand, one can imagine a phenomenological scenario in which one can create a cold dense matter through a heavy ion collision. In Fig. 9, one finds that the nuclear symmetry energy in the hadronic phase rises stiffly. The nuclear symmetry energy was calculated by using QCD sum rules, based mainly on linear density approximation [46]. The OPE terms and formula for plotting density the behavior can be found in Ref. [46]. Similar stiff density behavior of the symmetry energy was reported before in Ref. [4], including iso-spin dependent interaction channel ( $N\bar{L}\rho\delta$ ). If the symmetry energy increases stiffly at high density region, the neutron sea will become unstable. Then, it may lead to  $nn \rightarrow p\Delta^-$  type scattering. As the unstable neutrons are scattered out, the iso-spin asymmetry of remaining nuclear matter will be reduced. But, as proposed in Ref. [5, 6], if the mixed phase with quark and hadron appears, the iso-spin asymmetry can remain high due to the small symmetry energy of the quark phase: iso-spin distillation, by forming deconfined quark drop in the neutron. This distillation effect will cause enhanced  $\pi^-$  yields compared to the case where there is only hadronic phase. Our calculation shows enhanced quark matter symmetry energy when 2SC phase appears. So the distillation effect will be reduced and the ratio  $\pi^-/\pi^+$  will be smaller than the case where only the normal phase exists [5, 6]. Our consideration is in agreement with Ref. [7]. If the heavy ion collision experiments can be arranged to produce different temperature of dense matter, these two trend can be identified: one goes to the normal quark matter with low temperature and the other to the 2SC quark matter with extremely low temperature. The two experiment may give enhanced ratio  $\pi^-/\pi^+$  compared to the hadronic phase, but with a quite different value. Such experiment may be available at FAIR [51].

Although we have neglected strangeness degree of freedom for simplicity, considering its effects may give us a fruitful insight. Even if they are not the main excitation in the matter, one can expect high multiplicity of the hadron excitation containing strangeness. Measuring kaon multiplicity may be good probe for the cold dense matter as it has higher threshold energy and do not strongly interact with surrounding nuclear matter [47, 48]. Also the iso-spin dependence of hyperon mass and its interaction scattering cross section at high density regime can be an interesting issue. Describing these physics using explicit QCD quantum number may provide new understandings for high density iso-spin asymmetric matter. Related study is now in progress.

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